Section 2.2: Equilibrium solutions and stability

New vocabulary:

- phase diagram
- Autonomous first order d.e. $\frac{dx}{dt} = f(x)$, no t
- Critical points $\frac{dx}{dt} = 0$
- Equilibrium solution: constant solution where $\frac{dx}{dt} = 0$
- Stable, unstable

Stable: return to the solution from nearby points

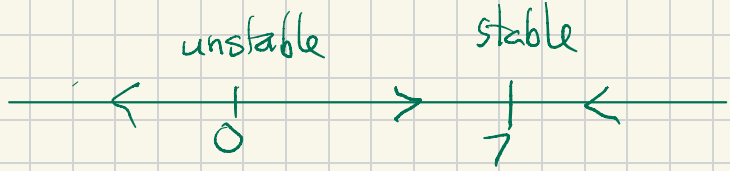
Unstable: we leave the solution from nearby points.

We don't need to know

- bifurcation point, bifurcation diagram, pages 91, 92, 93

We could have said something about the long term behavior of dx/dt = 4x (7-x) without solving the equation.

Phase diagram has critical points marked, arrows nearby to indicate the direction of the derivative.



Take $x = 6$: $\frac{dx}{dt} = 4 \cdot 6 \cdot 1 > 0$

$x = 8$ $\frac{dx}{dt} = 4 \cdot 8 \cdot (-1) < 0$

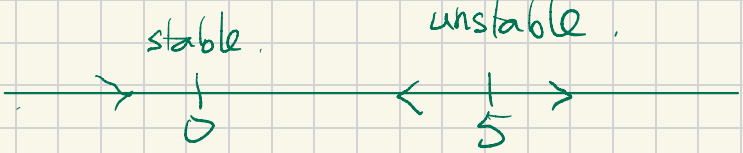
$x = -1$, $\frac{dx}{dt} = 4(-1)(7+1) < 0$

Questions 1 - 12

Consider an equation dx/dt = f(x)

Solve f(x) = 0 to find the critical points. Then analyze the sign of f(x) to determine whether each critical point is stable or unstable, and construct the phase diagram.

dx/dt = 3x (x-5)



Question. For dx/dt = x^2 (x-4)

1. How many critical points are there? 2
2. How many are stable? 0
3. How many are unstable 1

a. 1
b. 2
c. 3
d. 0

x = 0 is semistable.